



## Cambridge International AS & A Level

CANDIDATE  
NAME

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NUMBER

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### MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

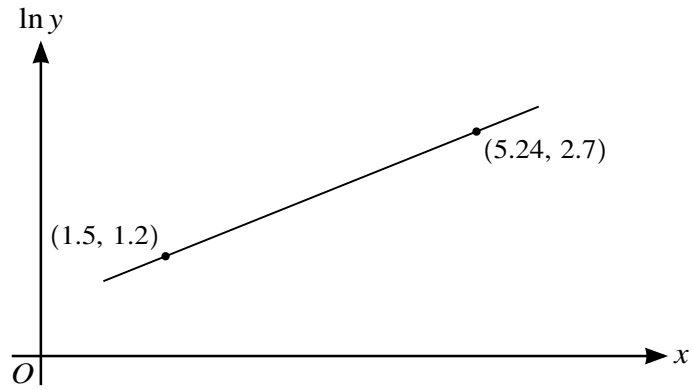
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.





2



The variables  $x$  and  $y$  satisfy the equation  $y^2 = Ae^{kx}$ , where  $A$  and  $k$  are constants. The graph of  $\ln y$  against  $x$  is a straight line passing through the points  $(1.5, 1.2)$  and  $(5.24, 2.7)$  as shown in the diagram.

Find the values of  $A$  and  $k$  correct to 2 decimal places. [5]

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3 Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$

[5]

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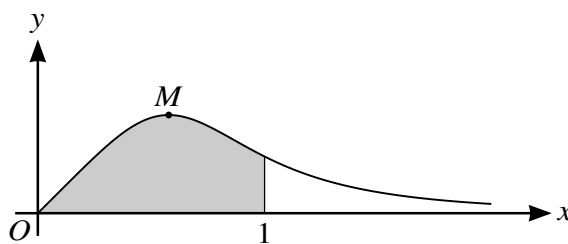
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The diagram shows the curve  $y = \frac{x}{1 + 3x^4}$ , for  $x \geq 0$ , and its maximum point  $M$ .

- (a) Find the  $x$ -coordinate of  $M$ , giving your answer correct to 3 decimal places. [4]

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- (b) Using the substitution  $u = \sqrt{3}x^2$ , find by integration the exact area of the shaded region bounded by the curve, the  $x$ -axis and the line  $x = 1$ . [5]

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7 The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = \frac{y - 1}{(x + 1)(x + 3)}$$

It is given that  $y = 2$  when  $x = 0$ .

Solve the differential equation, obtaining an expression for  $y$  in terms of  $x$ .

[9]

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8 (a) Solve the equation  $(1 + 2i)w + iw^* = 3 + 5i$ . Give your answer in the form  $x + iy$ , where  $x$  and  $y$  are real. [4]

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- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 2 - 2i| \leq 1$  and  $\arg(z - 4i) \geq -\frac{1}{4}\pi$ . [4]

- (ii) Find the least value of  $\text{Im } z$  for points in this region, giving your answer in an exact form. [2]

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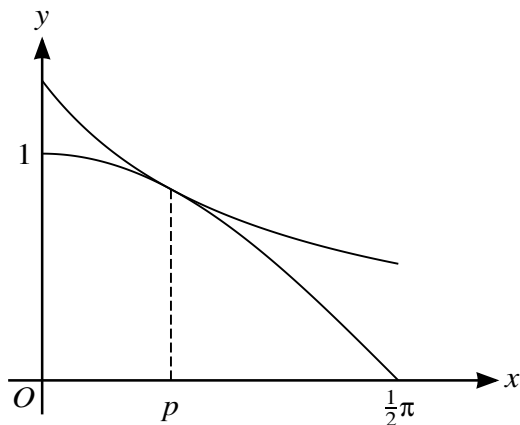
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The diagram shows the curves  $y = \cos x$  and  $y = \frac{k}{1+x}$ , where  $k$  is a constant, for  $0 \leq x \leq \frac{1}{2}\pi$ . The curves touch at the point where  $x = p$ .

- (a) Show that  $p$  satisfies the equation  $\tan p = \frac{1}{1+p}$ . [5]

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- (b) Use the iterative formula  $p_{n+1} = \tan^{-1}\left(\frac{1}{1+p_n}\right)$  to determine the value of  $p$  correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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- (c) Hence find the value of  $k$  correct to 2 decimal places. [2]

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The line through  $M$  and  $N$  intersects the line through  $O$  and  $B$  at the point  $P$ .

- (b) Find the position vector of  $P$ . [3]

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- (c) Calculate angle  $OPM$ , giving your answer in degrees. [3]

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**Additional Page**

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